Inferring market interest rate expectations from money market rates

By Martin Brooke of the Bank's Gilt-edged and Money Markets Division, and Neil Cooper and Cedric Scholtes of the Bank's Monetary Instruments and Markets Division.

The Bank's Monetary Policy Committee is interested in market expectations of future interest rates. Short-term interest rate expectations can be inferred from a wide range of money market instruments. But the existence of term premia and differences in the credit quality, maturity, liquidity and contract specifications of alternative instruments means that we have to be careful when interpreting derived forward rates as indicators of the Bank's repo rate. This article discusses the differences between some of the available instruments and relates these to the interest rate expectations that are calculated from them. It also describes the Bank's current approach to inferring rate expectations from these instruments.

Introduction

The Bank's Monetary Policy Committee (MPC) is interested in financial market participants' expectations of future interest rates. Knowledge of such expectations helps the MPC to predict whether a particular policy decision is likely to surprise market participants, and what their short-term response is likely to be to a given decision. Expectations of future levels of official rates also play a key role in determining the current stance of monetary policy. The Bank implements the MPC's monetary policy decisions by changing the level of its two-week repo rate which, in turn, influences the levels of other short-term money market interest rates. However, many agents in the economy are also affected by changes in longer-term interest rates. For instance, five-year fixed-rate mortgages are typically priced off the prevailing rates available on five-year swap contracts, and larger firms often raise finance in the capital markets by issuing long-maturity bonds. Changes in these longer-term interest rates depend to a considerable extent on expectations of future official rates. So the Bank needs to have some understanding of expectations of future policy rates, in order to monitor and assess changes in current monetary conditions.

The Bank performs the vast majority of its monetary operations via two-week sale and repurchase (repo) agreements—the Bank lends funds to its counterparties in return for specific types of collateral. Forward rates are the most commonly used measure of interest rate expectations. In principle, we want to derive forward rates that correspond to future two-week Bank repo rates. Unfortunately, however, there is no instrument that allows us to do this exactly. So we have to estimate forward rates from the sterling money market instruments that are actually traded. The Bank of England currently infers market interest rate expectations from: general collateral (GC) repo agreements; conventional gilt yields; interbank loans; short sterling futures contracts; forward-rate agreements (FRAs); and swap contracts settling on both the sterling overnight interest rate average (SONIA) and on six-month Libor rates. The box opposite explains how these instruments operate.

Other money market instruments such as certificates of deposit and commercial paper could also be used to derive forward rates. But the Bank does not use these instruments, as their credit quality can vary significantly from one issuer to the next. In contrast, interbank loans, short sterling futures, FRAs and Libor swaps all settle on Libor rates, determined by the British Bankers' Association (BBA). The credit risk element contained within each of these instruments will be common, and will be related to the financial institutions contained within the BBA's sample pool (see the box opposite). SONIA swap rates are likely to have very little credit risk as they embody expectations of movements in an overnight rate.

A range of maturities is available for each of the instruments outlined in the box, enabling us to calculate implied forward curves. However, the existence of term premia, arising from interest rate uncertainty and investor risk aversion, means that derived forward rates will not in general equal expectations of future short rates. Differences in credit quality, liquidity and contract specifications of the instruments also result in spreads between the forward curves. Consequently, none of these curves provides an unbiased measure of expectations of future official rates. Neither is any one instrument likely to provide consistently the best measure. So an understanding of the differences between the instruments is essential to assess market expectations of future monetary policy. This article explains why biases occur in measuring expectations and how the Bank takes them into account when trying to infer market participants' expectations of future official rates.

Sterling money market instruments

General collateral sale and repurchase agreements

Gilt sale and repurchase ('gilt repo') transactions involve the temporary exchange of cash and gilts between two parties; they are a means of short-term borrowing using gilts as collateral. The lender of funds holds government bonds as collateral, so is protected in the event of default by the borrower. General collateral (GC) repo rates refer to the rates for repurchase agreements in which any gilt stock may be used as collateral. Hence GC repo rates should, in principle, be close to true risk-free rates. Repo contracts are actively traded for maturities out to one year; the rates prevailing on these contracts are very similar to the yields on comparable-maturity conventional gilts.

Interbank loans

An interbank loan is a cash loan where the borrower receives an agreed amount of money either at call or for a given period of time, at an agreed interest rate. The loan is not tradable. The offer rate is the interest rate at which banks are willing to lend cash to other financial institutions 'in size'. The British Bankers' Association's (BBA) London interbank offer rate (Libor) fixings are calculated by taking the average of the middle eight offer rates collected at 11 am from a pool of 16 financial institutions operating in the London interbank market. The BBA publishes daily fixings for Libor deposits of maturities up to a year. A primary role of interbank deposits is to permit the transfer of funds from 'cash-surplus' institutions (such as clearing banks) to 'cash-deficit' institutions (those who hold financial assets but lack a sufficient retail deposit base).

Short sterling futures

A short sterling contract is a sterling interest rate futures contract that settles on the three-month BBA Libor rate prevailing on the contract's delivery date. Contracts are standardised and traded between members of the London International Financial Futures and Options Exchange (LIFFE). The most liquid and widely used contracts trade on a quarterly cycle with maturities in March, June, September and December. Short sterling contracts are available for settlement in up to six years' time, but the most active trading takes place in contracts with less than two years' maturity. Interest rate futures are predominantly used to speculate on, and to hedge against, future interest rate movements.

Forward-rate agreements (FRAs)

A FRA is a bilateral or 'over the counter' (OTC) interest rate contract in which two counterparties agree to exchange the difference between an agreed interest rate and an as yet unknown Libor rate of specified maturity that will prevail at an agreed date in the future. Payments are calculated against a pre-agreed notional principal. Like short sterling contracts, FRAs allow institutions to lock in future interbank borrowing or lending rates. Unlike futures contracts, which are exchange-traded, FRAs are bilateral agreements with no secondary market. FRAs have the advantage of being more flexible, however, since many more maturities are readily available. Non-marketability means that FRAs are typically not the instrument of first choice for taking speculative positions, but the additional flexibility does make FRAs a good vehicle for hedging, as they can be formulated to match the cash flows on outright positions.

Swaps

An interest rate swap contract is an agreement between two counterparties to exchange fixed interest rate payments for floating interest rate payments, based on a pre-determined notional principal, at the start of each of a number of successive periods. Swap contracts are, therefore, equivalent to a series of FRAs with each FRA beginning when the previous one matures. The floating interest rate chosen to settle against the pre-agreed fixed swap rate is determined by the counterparties in advance. There are two such floating rates used in the sterling swap markets: the sterling overnight interest rate average (SONIA) and six-month Libor rates.

SONIA is the average interest rate, weighted by volume, of unsecured overnight sterling deposit trades transacted prior to 3.30 pm on a given day between seven members of the Wholesale Money Brokers' Association. A SONIA overnight index swap is a contract that exchanges at maturity a fixed interest rate against the geometric average of the floating overnight rates that have prevailed over the life of the contract. SONIA swaps are specialised instruments used to speculate on or to hedge against interest rate movements at the very short end of the yield curve. Maturities traded in the market range from one week to two years.

Libor swaps settle against six-month Libor rates. They are typically used by financial institutions to help reduce their funding costs, to improve the match between their liabilities and their assets, and to hedge long positions in the cash markets. Traded swap contract maturities range from 2 years to 30 years.

Forward rates, the expectations hypothesis and term premia

Forward rates are the interest rates for future periods that are implicitly incorporated within today's interest rates for loans of different maturities. For example, suppose that the interest rate today for borrowing and lending money for six months is 6% per annum and that the rate for borrowing and lending for twelve months is 7%. Taken together, these two interest rates contain an implicit forward rate for borrowing for a six-month period starting in six months' time. To see this, consider a borrower who wants to lock in to today's rate for borrowing £100 for that period. He can do so by borrowing £97.08(1) for a year at 7% and investing it at the (annualised) six-month rate of 6%. In six months' time he receives back this sum plus six months' of interest at 6% (£2.92), which gives him the £100 of funds in six months' time that he wanted. After a year he has to pay back £97.08 plus a year of interest at 7% (£103.88). In other words, the borrower ensures that his interest cost for the £100 of funds he wants to borrow in six months' time is £3.88. He manages to lock in an interest rate—the forward rate(2) of 7.77% now for borrowing in the future.

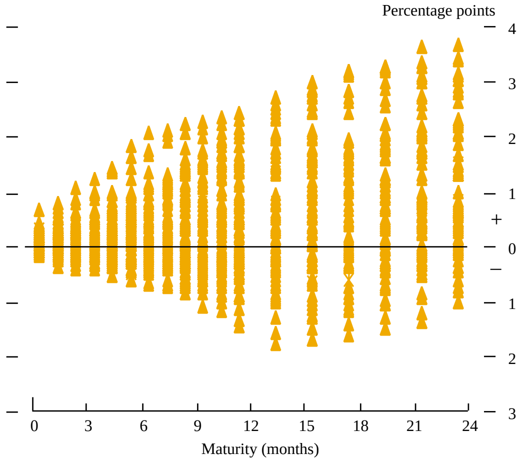
If there were no uncertainty about the path of future interest rates then forward rates would equal expected future interest rates. If this were not the case it would be possible to make unlimited riskless profits. Suppose, for example, that the borrower above knew for certain that six-month rates would be 8% in six months' time. But if today's six-month and twelve-month rates are 6% and 7%, then it is possible to lock in to borrowing now at 7.77%, knowing that one can then lend these funds out at a higher rate in six months' time to make a guaranteed riskless profit. Such an arbitrage opportunity would not persist long in a world of rational investors. As they exploited this situation, the configuration of interest rates would change until the implicit forward rates equalled expectations of future rates.

Future interest rates are, of course, not known with certainty. Nevertheless, if forward rates differ from expected future short rates, an investor will be able to create a position that has positive expected profits. The presence of interest rate uncertainty means that the actual profits from these trades may be positive or negative. Risk-averse investors will then require a risk premium to bear this interest rate risk. In equilibrium this will drive a wedge—the term premium—between the forward rate and expected short rates so that the expected profits incorporate the risk premium. Furthermore, the uncertainty surrounding the likely path of interest rates is greater the further ahead one looks, so this term premium is likely to increase with maturity. Hence the longer the horizon, the larger the difference between forward rates and expected rates.

Recent work at the Bank has tried to estimate the size of such term premia by comparing implied two-week interbank forward rates derived from a combination of Libor-related money market instruments with actual outturns of the Bank's two-week repo rate. If term premia are broadly stable, two-week interbank forwards should produce consistent forecast errors when regressed on the monetary policy rate outturns. However, consistent errors can also occur from repeated mistakes by market participants in forecasting the interest rate cycle. We can attempt to minimise this problem by comparing forward rates with subsequent Bank repo rates over a period spanning at least one complete interest rate cycle. If the sample period is sufficiently long, expectational errors should average out to zero. Any remaining bias should then represent the average term premium, though this technique will also pick up differences between the money market instruments used and the Bank's repo rate that are related to liquidity and credit quality.

Chart 1 plots the differences between our derived two-week interbank forward rates and the actual outturns of the official rate for alternative maturities out to two years, for the period January 1993 to September 2000. Each point represents the difference between the interbank forward rate and the corresponding outturn of the Bank's repo rate. It is clear from the chart that there is often a large degree of 'error' between the forward rate and the actual outturn. Unsurprisingly, the range of these errors increases with maturity, as it is harder to predict official rates further out. This dispersion also makes it hard to infer what the size of term premia are. The chart suggests that, on average, interbank forward rates have been biased above actual outturns of the official rate. The average biases over this period for six-month, one-year and two-year maturities were

Chart 1
Differences between two-week interbank forward rates and official rate outturns



(1) This is the present value of £100 in six months' time, $£100\left(\frac{1}{1+\frac{0.06}{2}}\right)$.

(2) The implicit forward rate is given by $2\left(\frac{(1+r_{0,12})}{\left(1+\frac{r_{0,6}}{2}\right)}-1\right)$ where $r_{0,\,12}$ is the one-year interest rate and $r_{0,\,6}$ is the six-month interest rate.

23, 45 and 109 basis points respectively. It should be noted, however, that these are forward rates derived from instruments that contain some element of credit risk. We estimate later in this article that credit risk considerations may account for 20–25 basis points, on average. The remainder of the bias observed in Chart 1 is due to either the existence of term premia or consistent expectational errors over the sample period. Given the volatility in the observed spread, we can draw only very tentative conclusions about the size of the term premia. Nevertheless, it seems reasonable to conclude that term premia create an upward bias in interbank forward rates compared with actual policy rate expectations, and that this bias increases with maturity.

Credit premia

As noted above, the Bank derives short-term forward interest rates from a variety of fixed-income instruments, which combine varying degrees of credit risk. GC repo is the closest instrument to the Bank's repo agreement. It is used by market participants for a number of purposes: it allows institutions to speculate about future changes in interest rates; retail banks use outright gilt holdings and GC repo to manage their day-to-day liquidity positions; and market-makers and other holders of gilts and gilt futures contracts can use the repo market to fund or close out their positions. Since the lenders of funds in the GC repo market are protected from default by the gilt collateral they hold, GC repo rates ought to be close to true risk-free rates and to the Bank's repo rate. In reality, however, GC repo tends to trade at rates below the Bank's repo rate for two-week maturities because of differences in liquidity and contract specifications between the Bank's and the GC repo agreements.

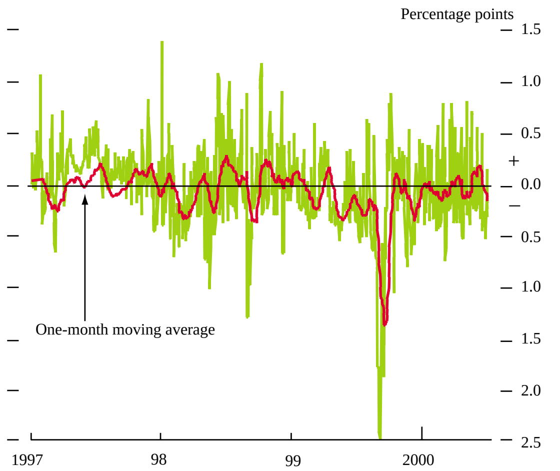
The measure of short-term interest rate expectations most frequently used by market participants is that derived from short sterling futures contracts. These settle at the three-month Libor rate prevailing on the contract's expiry. The implied future level of three-month Libor is simply a three-month forward rate. There are two difficulties in interpreting these forward rates as expectations of the Bank's repo rate. First, they indicate expectations for a three-month rate starting at the maturity of the contract. So they typically encompass three MPC decision dates and hence are an imprecise indicator of future two-week Bank repo rates. And second, Libor rates are based on uncollateralised lending within the interbank market and they consequently contain a credit premium to reflect the possibility of default. So expectations of future interbank rates will be higher than the Bank's repo rate.

Forward rates can also be derived from the term structures of both SONIA swaps and Libor swaps. The forward rates derived from Libor-based swaps will also include a credit risk premium. Just as for term premia, credit risk considerations are likely to increase with maturity. Since Libor swaps settle on six-month Libor, it is likely that the forward rates derived from these swaps will include a slightly larger credit risk bias than the forward rates derived from short sterling futures.

The fixed rate quoted for a SONIA swap represents the average level of SONIA expected by market participants over the life of the swap. SONIA usually follows the Bank's repo rate fairly closely because the credit risk on an overnight deposit is very low. The volatility of the spread between SONIA and the Bank's repo rate is large, however. This is an obvious reason for hedging using swaps. SONIA swaps are also used to take views about future changes in the Bank's repo rate (typically at maturities of between one and three months), and to speculate about market conditions that may drive short-term interest rates away from the official rate.

Chart 2 shows a time series of the spread between SONIA and the Bank's repo rate, and a simple expectation of the spread calculated as a one-month moving average. It shows that although the daily spread is highly volatile, the one-month 'expectation' is stable but often slightly below zero. This suggests that SONIA swaps should be a good indicator of rate expectations but with a small downward bias. Excluding December 1999 and January 2000 (which were affected by liquidity and credit risk considerations relating to the century date change), the spread has averaged -4 basis points since February 1997. This spread is most likely to reflect the trading practices of the principal money market participants, who need an upward-sloping yield curve between the overnight and three-month maturities in order to profitably undertake their market-making functions.

Chart 2
Spread of SONIA over Bank's two-week repo rate



Liquidity considerations

As noted above, differences between the forward rates derived from the various money market instruments may also reflect the different liquidity properties of the instruments. In general, market participants are often willing to pay a higher price (receive a lower yield) to hold instruments that are more liquid and that are likely to be easier to trade in distressed market conditions. There is no unique measure of liquidity, but turnover, market size, and bid-offer spreads may provide some indication of differing liquidity conditions.

Daily turnover in the gilt repo market is currently around £20 billion, with activity largely concentrated at the shortest end of the curve: 90% of the turnover matures between one and eight days, 6% at nine days to one month, and only 4% of turnover is at maturities of more than one month. Bid-offer spreads are typically around 5 basis points for most maturities. At the end of August, the total outstanding stock of gilt repo contracts was £133 billion.

The interbank deposit/loan market is slightly bigger, at around £160 billion. As with GC repo, activity is largely concentrated at maturities of less than one month, but market participants report that liquidity is reasonable out to three months. Bid-offer spreads vary depending on the borrower's creditworthiness but typically average around 3–5 basis points for three-month unsecured loans to high-quality borrowers.

Daily turnover in the short sterling futures market is currently around £45 billion and the total open interest in all contracts is around £385 billion. Contracts are very liquid in the first year and fairly liquid out to two years. Beyond that point, turnover is largely limited to arbitrage with the interest rate swap market and is often connected with hedging activity rather than speculation about future interest rates. Bid-offer spreads are generally 1–2 basis points for the first two years of short sterling contracts, and around 4 basis points after that.

Daily turnover in the SONIA swaps market is much smaller. The most liquid contract maturities are up to three months. Bid-offer spreads at these maturities tend to be around 2 basis points (ie about the same as short sterling).

So, with the exception of Libor-based swaps, all of the instruments are highly liquid in the very near term (ie out to one month). Then the differences become more apparent—gilt repo becomes less liquid after the one-month maturity range, SONIA swaps and interbank borrowing become less liquid after three months, while short sterling is less liquid after one to two years. Libor swaps are generally felt to be liquid in the two-year to ten-year maturity range. However, it is very difficult to quantify the impact of these differences in terms of the biases they are likely to produce in the forward rates derived from these instruments. Furthermore, liquidity conditions can change rapidly and so the biases are unlikely to be constant over time.

Other instrument-specific considerations

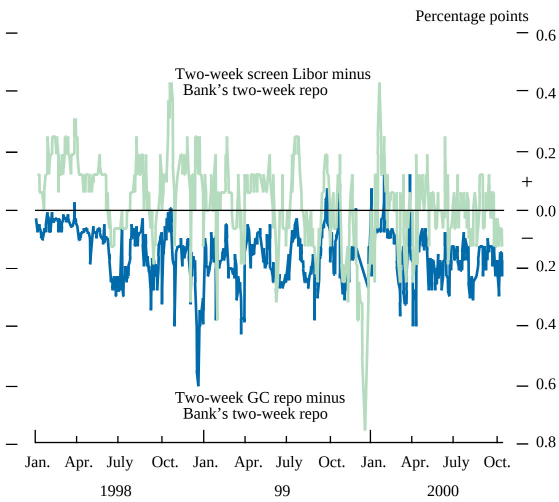
The Bank's two-week repo rate generally acts as a ceiling for the market-determined two-week GC repo rate. The reason for this is that if the market rate were to rise above the Bank's repo rate, counterparties to the Bank's open market operations would choose to borrow solely from the Bank of England, subject to the finite quantities of funding provided by the Bank. Two other specification differences between the Bank's two-week repo rate and the comparable-maturity GC repo rate add to this negative bias. First, the Bank allows its counterparties to replace one form of collateral with another during the life of the repo. This right of substitution, which is less common in market GC repo contracts, is potentially valuable to counterparties. Consequently, they are willing to lend collateral/borrow money from the Bank at a slightly higher interest rate. Around 13% of the collateral offered to the Bank in its open market operations is substituted for other collateral within the typical two-week lifetime of the repo transaction. Market participants believe that the right to substitution is worth around 3 basis points.

Another consideration is the fact that GC repo is used by the major retail banks to meet their liquidity requirements. This creates strong demand for short-dated gilts relative to the available supply. This, in turn, tends to tip the bargaining power in favour of holders of gilt collateral, enabling them to borrow cash at lower repo rates. In contrast, the Bank accepts a wider array of collateral in its repo operations. In particular, the range of eligible collateral for use in the Bank's repo transactions was expanded in August 1999 to include securities issued by other European governments (for which there is a much greater supply). Both of these considerations are likely to act in the same direction, putting downward pressure on two-week GC repo rates relative to the Bank's two-week repo rate.

How large are the biases?

How large are the biases due to credit, liquidity and the differences between Bank and GC repo? Chart 3 shows the spread between two-week GC repo and the Bank's repo rate. The spread has averaged close to -15 basis points and is highly volatile. The chart also shows the spread between

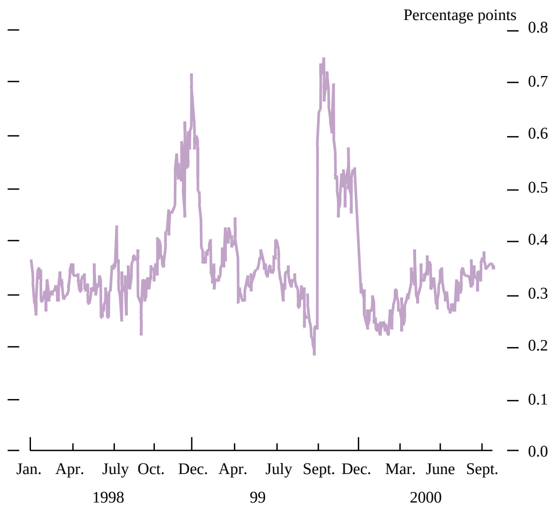
Chart 3
Two-week screen Libor and GC repo spreads against official rates



two-week Libor[1] and two-week Bank repo. This spread has averaged around 5 basis points, excluding December 1999 and January 2000, when the demand for secured borrowing increased sharply relative to unsecured borrowing because of credit concerns surrounding the century date change. This positive spread is likely primarily to reflect credit risk considerations between the unsecured interbank rate and the collateralised Bank repo rate. As noted previously, the credit risk premium contained within an interbank deposit will increase with its maturity—overnight lending is less risky than a three-month loan. So the credit risk contained within the forward three-month Libor rates derived from interbank loans, short sterling futures and FRAs is likely to be larger than this estimate. Similarly, swaps that settle on six-month Libor are likely to have a slightly larger credit risk element.

Chart 4 plots the spread between three-month Libor and three-month GC repo. Here, we are using the repo rate as an imperfect proxy for the riskless rate. In the run-up to the end of the year the spread widens. This effect is known as the 'year-end turn' and can be observed in a number of other markets. Excluding the three months at the end of the past two years, the average spread between the two rates has been around 35 basis points. Previously we noted that GC repo (at least at two-weeks' maturity) tends to be biased downwards compared with the Bank's repo rate. So around 15 basis points of this spread is likely to be related to the liquidity and contract differences discussed above. This

Chart 4
Three-month Libor minus three-month GC repo



leaves a credit spread of around 20 basis points between three-month Libor and the Bank's repo rate. Given the volatility of the spreads shown in Chart 2, it is important to recognise that these estimates are averages and that the differences between the forward rates derived from these instruments will vary over time.

Assessing near-term interest rate expectations

Given the observed level and behaviour of the spreads we can attempt to make a judgment about market expectations of the Bank's repo rate. The Bank's approach follows three stages:

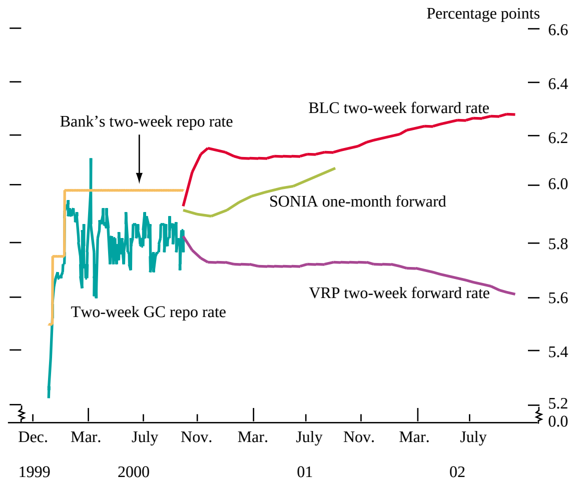
- we estimate two alternative forward curves from two alternative sets of instruments, each with common credit risk characteristics;
- we adjust these forward curves for the biases created by credit, liquidity and contract specification differences; and
- finally, we take a view on the adjustment required to take into account the bias introduced by the existence of term premia.

Both our estimated curves use the Bank's variable roughness penalty (VRP) curve-fitting technique explained in Anderson and Sleath (1999).[2] The first curve is fitted to GC repo rates up to six months and to gilt yields of greater than three months' maturity. The yields on comparable-maturity GC repo contracts and conventional gilts are very similar. Hence this combination of instruments does not introduce any discontinuity into the fitted forward curve. The front three to six months of the forward curve is largely influenced by the GC repo data and after this the forward curve reflects the influence of the conventional gilts. The second forward curve is an estimated two-week 'bank liability curve' (BLC). This is a curve fitted to synthetic bond prices generated from a combination of instruments that all settle on Libor rates. The instruments used are BBA interbank offer rates, short sterling futures, FRAs and, beyond two years, interest rate swaps. (The synthetic bond construction and curve-fitting processes are described in more detail in the appendix on pages 400–02.) The front twelve months of this curve is largely dependent on the interbank offer rates, FRAs and short sterling futures, while the next year is mainly influenced by short sterling futures and FRAs. Beyond two years, Libor swaps are the dominant influence. Chart 5 shows both forward curves, as well as a simple series of one-month forward rates derived from the available quoted rates for different-maturity SONIA swaps.

To interpret the curves in Chart 5 as indications of market expectations of future short rates we next need to adjust for the different types of bias discussed above. It is useful to do this in stages: first consider what a true risk-free forward curve corresponding to the Bank's two-week repo rate would look like, taking into account the credit risk biases in the bank liability curve and the downward bias of GC repo; and second to adjust for the term premia that exist within any forward curve. Because we have limited data on how

(1) This data is collected by the Bank from brokers rather than from the BBA.
(2) See Anderson, N and Sleath, J (1999), 'New estimates of the UK real and nominal yield curves', *Bank of England Quarterly Bulletin*, November, pages 384–96. The appendix on pages 400–02 gives a brief outline of the VRP technique.

Chart 5
Forward rates with historic two-week GC repo(a)



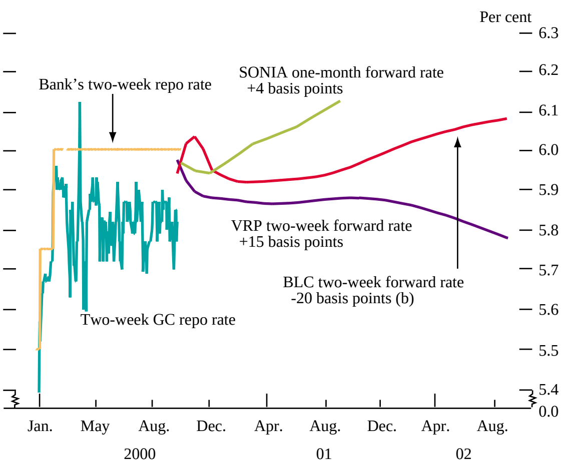
(a) As at 27 October.

these spreads vary at different maturities we can make only simple rough and ready adjustments.

The downward bias in two-week GC repo is approximately 15 basis points, so we can adjust the front end of the VRP gilt curve upwards by this amount to get our estimate of the 'Bank repo' forward curve. Likewise, the bank liability curve needs to be adjusted down by 5 to 10 basis points at the first month or so, rising to 20 basis points from three months to two years. Beyond two years, the bank liability curve is primarily influenced by swaps settling on six-month Libor rates and so the credit risk element is likely to rise to around 25 basis points. The forward rates derived from SONIA swaps need to be adjusted upwards by 4 basis points.

These adjusted curves are shown in Chart 6. Using money market rates prevailing on 27 October, the starting-points for all three of the forward curves were below the Bank's repo rate, even after making our adjustments. This reflects the

Chart 6
Adjusted forward rates with historic two-week GC repo(a)



(a) As at 27 October.
(b) Adjustment of 5 basis points at two weeks, growing to 20 basis points at two months and beyond.

volatility of the spreads between the market rates and the Bank's repo rate; we have been able to adjust only for the average observed premia. For the first year, the gilt and bank liability curves were telling a consistent story—both were broadly flat and suggested that the market's mean expectation was for no change in rates over the next year. In Section 6 of the *Inflation Report*, the Bank presents projections of inflation and GDP based on market interest rate expectations. The current convention is to use the adjusted GC repo/gilt forward curve as in Chart 6 to estimate these expectations.

Beyond a year, however, these two curves diverge. This is puzzling, as we have taken into account (albeit in a simple way) the differences between the forward curves due to credit risk. Term premia effects have not been allowed for in Chart 6, but these are likely to influence all the derived forward rates in the same way and so are unlikely to explain the divergence. One potential explanation is that short sterling futures rates are biased upwards because the demand to hedge against the possibility of higher interest rates exceeds the demand to hedge against the chance of lower rates. Hedging against the possibility of higher interest rates in the future involves the creation of a short position in futures contracts. If interest rates rise in the future, the price of these contracts will fall making the hedge position profitable. This hedging activity (ie selling short sterling contracts) may be pushing up short sterling futures rates to higher levels than they would otherwise be. An alternative explanation is that the low issuance of short-maturity gilts by the UK government has led to their yields, and the forward rates associated with them, being depressed compared with the true risk-free rates.

Finally we need to take into account the effects of term premia. We have only the simple estimates discussed earlier, which suggest that term premia were negligible at less than six months and thereafter suggest a downward revision to the forward curves. Given this information, the forward rates derived from all the sterling money market instruments implied an expectation that the MPC would not raise the Bank's repo rate in the next two years.

Conclusions

In summary, this article has argued that:

- Forward rates estimated from money market instruments are biased estimates of expectations of future Bank repo rates because of term, credit and liquidity premia, as well as contract specification differences.

- No particular money market instrument is likely to provide a 'best' indication of Bank repo rate expectations at all maturities. The spreads between the Bank's two-week repo rate and the instruments used to estimate our market curves are volatile and so we cannot expect to get a result that is common across all instruments.

- Reflecting these considerations, the Bank estimates two forward curves: one employing GC repo and gilt data and one that uses a combination of sterling money market instruments that settle on Libor rates.

- A number of simple ready-reckoner adjustments can be applied to the two estimated forward curves in an attempt to transform them into an estimate of a forward curve equivalent to two-week Bank repo rates. First, the GC repo/gilt forward curve needs to be adjusted up by around 15 basis points and the bank liability curve adjusted down by around 20 basis points. After these changes we still need to consider the impact of term premia effects. Preliminary estimates suggest that this would require us to make a further downward adjustment to both curves beyond a six-month horizon. However, we currently have limited information on the size of the term premia that create biases in forward curves even after we have taken into account estimates of credit and liquidity premia.

Appendix
Estimating a 'bank liability' forward curve using the Bank's VRP curve-fitting technique

The Bank has recently developed a method of estimating a yield curve from interbank liabilities. The new bank liability curve (BLC) uses sterling money market instruments that settle on Libor to construct synthetic 'interbank bonds'. The prices of these synthetic bonds are then used to fit a unified forward curve using the Bank's VRP curve-fitting technique.

Constructing synthetic bank liability bonds

Conceptually, the main issue is how to convert money market and swap market instruments into synthetic bonds. The bank liability instruments used in our curve are:

- interbank loan rates (represented by BBA Libor fixings);
- short sterling futures;
- forward-rate agreements; and
- Libor-based interest rate swaps.

The common thread linking all these instruments—which permits us to estimate a unified forward curve from their rates—is that they are referenced on BBA Libor fixings. This ensures that the instruments are generally comparable in terms of underlying counterparty credit risk, in the sense that they can be treated as if issued by a 'representative' high-quality financial institution.

Interbank loans

An interbank loan is, in effect, a zero-coupon bond. The Libor fixing rate therefore relates to the price of a synthetic zero-coupon bond as follows:

$$B_L(t_0,t_n) = \frac{100}{1+L(t_0,t_n)\alpha(t_0,t_n)} \quad \text{where} \quad \alpha(t_0,t_n) = \frac{t_n - t_0}{36,500}$$

where $B_L(t_0, t_n)$ is the price at t_0 for a synthetic zero-coupon Libor-based bond of maturity t_n; $L(t_0, t_n)$ is the annualised Libor deposit rate at t_0 for maturity date t_n; and $\alpha(t_0, t_n)$ is the day-count basis function for sterling Libor loans and deposits.

Forward-rate agreements

Purchasing a forward-rate agreement (FRA) allows an investor to transform, at time t_0, a floating-rate liability commencing at t_m and maturing at t_n into a fixed-rate liability. It achieves this by paying out the difference between a reference floating rate and the pre-specified FRA rate on a notional amount. If the reference rate turns out to be above the FRA rate, the investor would then receive payment on the FRA contract, and this payment would exactly offset the higher costs of a floating-rate loan with the same principal. The end-product would be a fixed-rate loan set at the FRA rate, commencing at t_m and ending at t_n (a forward-start fixed-rate loan). Combining a fixed-rate Libor deposit maturing at t_m with a forward-start fixed-rate loan (constructed as above) commencing at t_m and maturing at t_n thereby gives a synthetic zero-coupon bond with maturity t_n.

A useful property of a $(t_m \times t_n)$ FRA is that the contract commences on the same date as the matching t_m Libor deposit expires, and ends on the same date as the t_n Libor deposit expires. Correspondingly, the end of one FRA contract coincides with the beginning of the next. For underlying contract start dates twelve months or less into the future, the price of a synthetic Libor/FRA zero-coupon bond would be given by:

$$B_{FRA}(t_0,t_n) = \frac{B_L(t_0,t_m)}{1+f_{FRA}(t_0,t_m,t_n)\alpha(t_m,t_n)}$$

where $\alpha(t_m,t_n) = \dfrac{t_n - t_m}{36,500}$

and $f_{FRA}(t_0, t_m, t_n)$ is the FRA rate commencing at t_m and ending at t_n. For FRA contracts commencing beyond twelve months (the longest Libor rate) we can construct synthetic bonds by combining FRAs in a similar way. Hence for $t_0 < t_l < t_m < t_n$, where $t_l \leq$ twelve months and $t_m >$ twelve months:

$$B_{FRA}(t_0,t_n) = B_L(t_0,t_l)\left[\frac{1}{1+f_{FRA}(t_0,t_l,t_m)\times\alpha(t_l,t_m)}\right] \times \left[\frac{1}{1+f_{FRA}(t_0,t_m,t_n)\times\alpha(t_m,t_n)}\right]$$

Longer-term bond prices may be calculated in the same way using additional FRAs.

Short sterling futures (SSFs)

A difficulty arises when considering SSFs because futures contract dates will in general not coincide with Libor expiry dates, and some of the futures contracts will commence beyond the longest Libor deposit contract. For SSFs commencing less than twelve months ahead, the same approach as for FRAs can be used to obtain synthetic Libor/SSF zero-coupon prices. But we need a Libor-based bond price that matures at the maturity of the short sterling future. To calculate this we linearly interpolate across Libor rates to get an estimate of the bond price $\hat{B}_L(t_0,t_m)$ that matures at the same time, t_m, as the futures contract.

Hence synthetic zero-coupon Libor/SSF 'bond' prices would be given by:

$$\hat{B}_{SSF}(t_0,t_n) = \frac{\hat{B}_L(t_0,t_m)}{1 + f_{SSF}(t_0,t_m,t_n)\alpha(t_m,t_n)}$$

where $\alpha(t_m,t_n) = \frac{t_n - t_m}{36,500}$

and f_{SSF} is the short sterling futures rate maturing at t_m.

Beyond twelve months, it becomes necessary to bootstrap futures contracts together. This requires us to assume that the SSFs have an underlying interbank loan contract with the same term as the time to the next contract, to ensure strip continuity. Fortunately, day-count errors will matter proportionately less at longer maturities.(1) We can then bootstrap the futures onto the latest available (interpolated) Libor discount factor.

The bootstrapped bond prices can be obtained as follows:

$$\hat{B}_{SSF}(t_0,t_n) = \hat{B}_L(t_0,t_m)\prod_{j\geq m}^{n}\left(\frac{1}{1 + f_{SSF}(t_0,t_j,t_{j+1})\alpha\left(t_j,t_{j+1}\right)}\right)$$

where t_j (j = 1, … , J) represents the SSF contract dates and t_m is the start-date for the last SSF contract commencing within twelve months.

Interest rate swaps

A par swap can be thought of as a portfolio of fixed-rate and floating-rate cash flows. For the purchaser of a par swap of maturity t_N, the fixed leg of the swap involves a series of outgoing interest payments on a notional principal at a predetermined fixed swap rate, $s(t_0, t_N)$. The floating leg involves incoming interest payments on the same notional principal, but linked to a floating reference rate, reset at given intervals (usually six-month Libor for sterling swaps). A par swap is an interest rate derivative with zero initial premium—ie the swap rate, $s(t_0, t_N)$, is set such that the fixed and floating 'legs' of the swap have equal present value. The present value of the floating leg is £1. Hence equating the fixed and floating legs gives:

$$1 = s(t_0,t_N)\sum_{n=1}^{N}\alpha(t_{n-1},t_n)B(t_0,t_n) + B(t_0,t_N)$$

where $\alpha(t_0, t_n)$ is the day count function and $B(t_0, t_n)$ is the price of a zero-coupon bond with face value £1 and maturity t_n. The swap rate, $s(t_0, t_N)$, can be interpreted as the coupon rate, payable at the payment dates t_n (n = 1, … , N), giving the coupon bond a market price at t_0 equal to its face value.

Typically, swap counterparties exchange the net difference between fixed-rate and floating-rate obligations at the 'coupon' dates. However, we use the formula to calculate the 'fixed-rate coupon' payable on the synthetic fixed-rate bond trading at par.(2) Once refixing and settlement dates are determined, interest payments are calculated using the standard formula:

$$INT = P \times R/100 \times \alpha(t_{n-1}, t_n)$$

where $\alpha(t_{n-1}, t_n) = (t_n - t_{n-1})/365$; P is the nominal principal; R is the fixed/floating rate (annualised but with semi-annual compounding); t_n is the settlement date n = 1, … , N; and $\alpha(t_{n-1}, t_n)$ is the day-count fraction (actual/365(fixed) for sterling swaps).

Transforming bank liability instruments into synthetic zero-coupon and coupon bonds in this fashion allows one to build a bond price vector and a simple cash-flow matrix. Applying the Bank's existing curve-fitting technique then yields a forward curve for bank liabilities.

Fitting the forward curve

The Bank currently fits a forward curve through bond price data using spline-based techniques model forward rates as a piecewise cubic polynomial, with the segments joined at 'knot-points'. The coefficients of the individual polynomials are restricted such that both the curve and its first derivative are continuous at all maturities, including the knot-points. The Bank's approach involves fitting a cubic spline by minimising the sum of squared price residuals plus an additional roughness penalty.

To be more precise, the objective is to fit the instantaneous forward rate, $f(m)$, to minimise the sum of squared bond price residuals weighted by inverse modified duration, plus an additional penalty for 'roughness' or curvature, weighted according to maturity. In the Bank's specification, the roughness penalty, $\lambda_t(m)$—which determines the trade-off between goodness of fit and the smoothness of the curve—is a function of maturity, m, but is constant over time, t. This allows the curve to have greater flexibility at the short end. Weighting bond price errors by inverse duration gives approximately equal weight to a fractional price error across all maturities.

The objective function to be minimised is:

$$X = X_P + \int_0^N \lambda_t(m)\left[f''(m)\right]^2 dm$$

(1) Typically, SSFs are spaced 91 days apart, though they can be as much as 98 days apart. The term of the underlying three-month Libor contract will usually differ from this.

(2) Note the contrast between coupons on synthetic bank bonds and gilts. Gilts pay out a coupon determined by the formula: $INT = P \times R \times {}^1/_2$, regardless of the precise day on which the coupon falls. Gilts therefore have 'fixed' coupons, whereas synthetic bank bonds have 'fixed-rate' coupons, the size of which depend on the day-count since the previous coupon.

where $X_P = \sum_{i=1}^{I}\left[\frac{P_i - \prod_i(\beta)}{D_i}\right]^2$

and $f(m)$ is the instantaneous forward rate for maturity m, P_i and $\Pi_i(\beta)$ are the observed and fitted bond prices respectively, and β is the vector of parameters. The parameters to be optimised are the parameters of the smoothing function, $\lambda(m)$, and the number of knot-points. The smoothing function is specified as follows:

$$\log\lambda(m) = L - (L-S)\exp(-m/\mu)$$

where L, S and μ are parameters to be estimated, as explained in Anderson and Sleath (1999).